

# Basic study of cross coupling canceling in power transmission by two pairs of coils using magnetic resonance coupling

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**Abstract**—Regarding magnetic resonance coupling, which is highly useful for wireless power transfer that enables high efficiency, long transmission distance, and tolerance of misalignment, coils are used when considering power transmission by providing multiple coils on the power transmission side or power reception side. We generalized the effects of cross coupling on transmission efficiency and power, and verified the improvement of characteristics by cross coupling canceling. In power transmission with two power-receiving coils for two power-transmitting coils, cross coupling canceling, or elimination of canceling a particular magnetic coupling, is equivalent to manipulating the coupling elements in the theoretical equations for transmission efficiency and transmission power. It is shown by circuit simulation and experiment.

**Keywords**—wireless power transfer, multiple feed, magnetic resonance coupling, cross coupling, cross coupling canceling

## I. INTRODUCTION

Magnetic resonance coupling, which has recently emerged as an application for wireless power transfer (WPT), has advantages over conventional electromagnetic induction systems, such as relatively long transmission distances and less efficiency loss due to misalignment, etc. In the field of developing WPT, in-car power transmission for EVs has been developed, and these coils have It is required to generate a strong local magnetic flux with relatively high flux levels and to ensure efficient magnetic field coupling not only to generate horizontal flux but also to ensure efficient magnetic field coupling even when the gap between the charging coil and the EV coil in the ground or on the surface is large. It is necessary to allow for misalignment in the direction.

The coupling between closely adjacent coils in multiple feeds, which affects transmission efficiency and so on, occurs; in a WPT system, not only one-to-one, but also one-to-many [1][2][3][4][5] or multiple-to-many [6] coils are possible. In this case, cross coupling (CC) occurs between the relay coils and multiple transmission and receiving coils. It has been reported that the presence or absence of CC causes a change in the frequency response in power transmission [7], frequency tracking with CC [8] and impedance matching with CC [9] have been reported to maximize the efficiency of the transmission coils. For CC described in these reports, an efficiency formulation considering CC in the case of one-to-two transmitter/receiver coils and a method for improving efficiency by insertion of canceling coils and canceling capacitors [10] are also presented. However, most of the reports so far have discussed the efficiency subject, and the

transmission power considering CC has not been considered. The effect of CC needs to be studied clearly by broadening the base to include transmission power as well as efficiency.

This paper proposes an efficiency and power formulation with an equivalent circuit that takes into account CC between the coils and evaluates the changes in efficiency and power characteristics by canceling the CC between the coils in a configuration with two transmission coils and two receiving coils with minimum units of CC. In Chapter 2 is the process of formulating the efficiency and transmission power of multiple power supplies, Chapter 3 examines the effects of cross coupling canceling (CCC) based on the parameters of the developed experimental equipment, and Chapter 4 presents a summary.

## II. THE PROCESS OF FORMULATION

### A. System overview and equivalent circuit

Fig.1 shows the overall picture of the transmission system: the coupling between Tx and Rx is called the main coupling and the coupling between Tx and Tx or Rx and Rx is called CC. In the equivalent circuit shown below, the main coupling is shown as  $L_m$  and CC between Tx and Tx or Rx and Rx as  $L_c$ . This section deals with the phenomenon including the effect of CC in power transmission using two pairs of coils, which corresponds to the case where there are multiple coils on both the EV and ground sides and multiple coils on the ground side are in the power-on state at the same time. When the Tx and Rx coupling is minimal, in the above application CC can be ignored. However, if the Tx-Tx or Rx-Rx coupling is strong, the impact on efficiency and transmission power needs to be visualized.

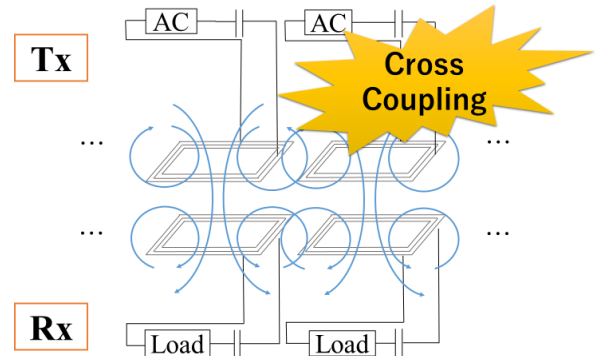


Fig.1. General view of the transmission system

### B. Derivation of Multiple Power Supply Formula

In this section, we derive the equations for the efficiency of power transmission and input/output power at multiple

loads... We note that CC can be positive or negative in some cases [10]. The formulation in this paper considers the case that CC has a negative value, i.e., when a coil is used for CCC. Fig.2 shows the equivalent circuit of the transmission system shown in Fig.1. The relationship between self-inductance and mutual inductance and the voltage and current of each device, taking into account CC at all points, is shown in equation (1). Here, the reactance due to the self-inductance  $L_n$  and the capacitance  $C_n$  cancels out theoretically when the operating frequency is  $\omega_0$  (see the equation (2)), and the power supply voltage of Tx is set to be in opposite phase to each other to avoid complications of the general equation to be derived somewhat. The following conditions are applied (see equation (3)).

- The internal resistance and load resistance in the resonator are the same between the Tx and Rx coils.
- The coupling between each Tx coil and the Rx coil corresponding to each Tx coil and the Rx coil in the diagonal direction of each Tx coil is symmetrical.

Equation (4) is a relational equation applying the conditions in equations (2) and (3). The input power can be considered equivalent to the sum of the power consumed by the transmission and receiving devices, and the input power  $P_1$  and the output power  $P_3$  of the transmission and receiving devices in a pair are shown in equation (5). Although two pairs of coils are used in this study, here we look at the transmission efficiency in one pair (see equation (6)).

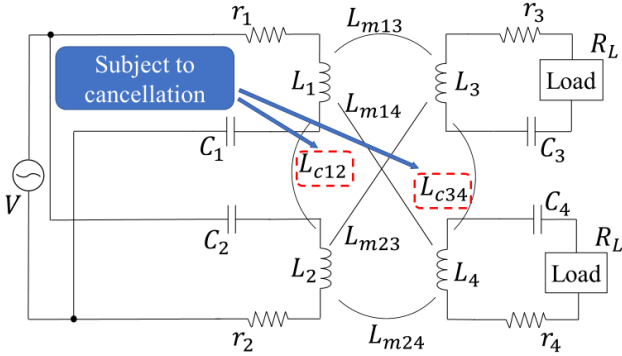


Fig.2. Equivalent circuit of the transmission system

$$\begin{bmatrix} V \\ V \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_1 & -j\omega L_{c12} & j\omega L_{m13} & j\omega L_{m14} \\ -j\omega L_{c12} & r_2 & j\omega L_{m23} & j\omega L_{m24} \\ j\omega L_{m13} & j\omega L_{m23} & r_3 + R_L & -j\omega L_{c34} \\ j\omega L_{m14} & j\omega L_{m24} & -j\omega L_{c34} & r_4 + R_L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \quad (1)$$

$$j\omega L_n + \frac{1}{j\omega C_n} = 0 \quad (2)$$

$$\begin{cases} r_1 = r_2 \\ r_3 = r_4 \\ R_3 = R_4 = R_L \\ L_{m13} = L_{m24} = L_m \\ L_{m14} = L_{m23} = L_{dia} \end{cases} \quad (3)$$

$$\begin{bmatrix} V \\ V \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_1 & -j\omega L_{c12} & j\omega L_m & j\omega L_{dia} \\ -j\omega L_{c12} & r_1 & j\omega L_{dia} & j\omega L_m \\ j\omega L_m & j\omega L_{dia} & r_3 + R_L & -j\omega L_{c34} \\ j\omega L_{dia} & j\omega L_m & -j\omega L_{c34} & r_3 + R_L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \quad (4)$$

$$\begin{cases} P_1 = P_{r_1} + P_{r_3} + P_{R_L} = r_1 |I_1|^2 + r_3 |I_3|^2 + R_L |I_3|^2 \\ P_3 = P_{R_L} = R_L |I_3|^2 \end{cases} \quad (5)$$

$$\eta = \frac{P_3}{P_1} = \frac{R_L |I_3|^2}{r_1 |I_1|^2 + r_3 |I_3|^2 + R_L |I_3|^2} \quad (6)$$

Next, we derive the transmission characteristics formula for the case of CCC. When canceling coils with a reactance of self-inductance  $L_{Tcan}$  and  $L_{Rcan}$  are inserted in series in each of the circuits on the Tx and Rx side, the resonant condition before the insertion of the coils also applies. In this case, the equation of voltage and current is as shown in equation (7).

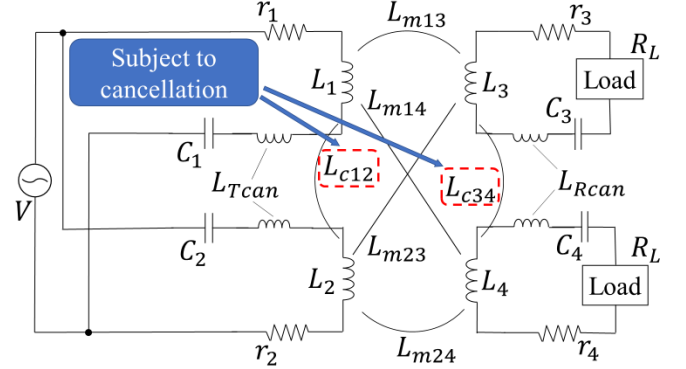


Fig.3 Equivalent circuit of the transmission system when inserting the canceling coil

### C. Comparison of Efficiency and Power Theoretical Equations with and without CCC

The comparison of the results with and without CCC is shown in TABLE.1 and 2. It is possible to improve the efficiency up to the case where CC can be neglected by eliminating the term of coefficients on  $L_{c34}$ . On the other hand, TABLE.2 shows that the elements  $L_{Tcan}$  and  $L_{Rcan}$  due to the canceling coil are shown only in the terms of  $L_{c12}$  and  $L_{c34}$  in the power equation. As well as the efficiency, theoretically, the primary input power  $P_1$  and the secondary output power  $P_3$  can be improved to the power values where CC can be neglected by eliminating the coefficients of the terms  $L_{c12}$  and  $L_{c34}$ , depending on the values of  $L_{c12}$  and  $L_{c34}$ .

## III. BASIC EVALUATION OF CROSS COUPLING CANCELING

### A. Consistency between formulation and Experiment

In this section, CCC is evaluated by substituting the parameters of the fabricated coils into the theoretical equations of efficiency and power derived in Chapter 2. In the transmission circuit system, the input power supply  $V = 1$  V, the Rx load  $R_L = 2 \Omega$  is fixed, and the resonant frequency is 45 kHz for both the transmission and receiving sides. In this paper, we compare the parameters of the devices for which the resonance and CCC of each circuit can ideally be achieved (Ideal Param.) with those of the measured devices (Exp. Param.). The ideal device for both circuits is

$$\begin{bmatrix} V \\ V \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_1 + j\omega L_{Tcan} & -j\omega L_{c12} & j\omega L_m & j\omega L_{dia} \\ -j\omega L_{c12} & r_1 + j\omega L_{Tcan} & j\omega L_{dia} & j\omega L_m \\ j\omega L_m & j\omega L_{dia} & r_3 + R_L + j\omega L_{Rcan} & -j\omega L_{c34} \\ j\omega L_{dia} & j\omega L_m & -j\omega L_{c34} & r_3 + R_L + j\omega L_{Rcan} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \quad (7)$$

TABLE.1 Comparison of Efficiency

	Efficiency : $\eta$
Without CCC	$\frac{(L_m - L_{dia})^2 R_L \omega^2}{\{(L_m - L_{dia})^2 (r_3 + R_L) + r_1 L_{c34}^2\} \omega^2 + r_1 (r_3 + R_L)^2}$
With CCC	$\frac{(L_m - L_{dia})^2 R_L \omega^2}{\{(L_m - L_{dia})^2 (r_3 + R_L) + r_1 (L_{Rcan} - L_{c34})^2\} \omega^2 + r_1 (r_3 + R_L)^2}$

TABLE.2 Comparison of Power

	Power (Primary side) : $P_1$
Without CCC	$\frac{V_1^2 \{(L_m - L_{dia})^2 (r_3 + R_L) + r_1 L_{c34}^2\} \omega^2 + r_1 (r_3 + R_L)^2}{\{(L_m - L_{dia})^2 - L_{c12} L_{c34}\} \omega^2 - \gamma^2 + \{r_1 L_{c34} + L_{c12} (r_3 + R_L)\}^2 \omega^2}$
With CCC	$\frac{V_1^2 \{(L_m - L_{dia})^2 (r_3 + R_L) + r_1 (L_{Rcan} - L_{c34})^2\} \omega^2 + r_1 (r_3 + R_L)^2}{\{(L_m - L_{dia})^2 + (L_{c12} - L_{Tcan})(L_{Rcan} - L_{c34})\} \omega^2 - \gamma^2 - \{r_1 (L_{Rcan} - L_{c34}) - (L_{c12} - L_{Tcan})(r_3 + R_L)\}^2 \omega^2}$
	Power (Secondary side) : $P_3$
Without CCC	$\frac{V_1^2 (L_m - L_{dia})^2 R_L \omega^2}{\{(L_m - L_{dia})^2 - L_{c12} L_{c34}\} \omega^2 - \gamma^2 + \{r_1 L_{c34} + L_{c12} (r_3 + R_L)\}^2 \omega^2}$
With CCC	$\frac{V_1^2 (L_m - L_{dia})^2 R_L \omega^2}{\{(L_m - L_{dia})^2 + (L_{c12} - L_{Tcan})(L_{Rcan} - L_{c34})\} \omega^2 - \gamma^2 - \{r_1 (L_{Rcan} - L_{c34}) - (L_{c12} - L_{Tcan})(r_3 + R_L)\}^2 \omega^2}$

$$\gamma = -r_1 (r_3 + R_3)$$

TABLE.3 Parameters of fabricated devices

		Tx		Rx	
		$n = 1$	$n = 2$	$n = 3$	$n = 4$
Main Coils	$L_n$ [ $\mu$ H]	47.4	48.8	48.6	46.4
	$r_n$ [ $\Omega$ ]	0.093	0.089	0.074	0.075
	$C_n$ [nH]	261.8	249.7	251.8	268.2
Cancel Coils	$L_n$ [ $\mu$ H]	5.62	5.61	5.80	5.50
	$r_n$ [ $\Omega$ ]	0.0089	0.019	0.0084	0.012

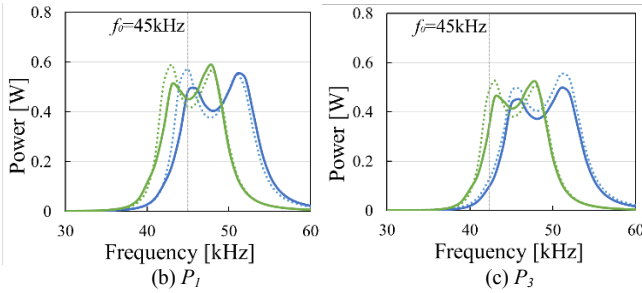
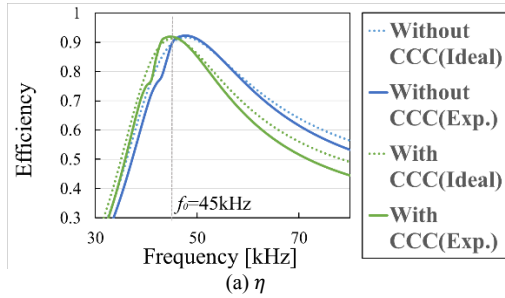


Fig.4 Consistency between formulation and circuit analysis

$L_n = 50 \times 10^{-6} \mu\text{H}$ ,  $r_n = 0.1 \Omega$  and  $C_n = 250 \times 10^{-9} \text{nH}$ . When the CCC is achieved, the terms for  $L_{c12}$  and  $L_{c34}$  in TABLE.1 and 2 are assumed to be zero. The parameters of

the fabricated coils are shown in TABLE.3. They were measured with impedance analyzer (KEYSIGHT E4900A). The self-inductance, internal resistance and capacitance of the resonant capacitor of each coil are denoted as  $L_n$ ,  $r_n$  and  $C_n$ , respectively. The parameters of the canceling coil inserted into each circuit for CCC are also shown. Fig.4 shows the comparison between the transmission characteristics of the ideal device (Ideal.) and the circuit analysis results using measured parameters (Exp.). It can be confirmed that the peak efficiency and the frequency point for the minimum power are almost the same for both devices. Although the values at arbitrary frequencies are different between the formulations and the experiment, the motion of the characteristics are almost the same.

### B. Relative value change of CC

We have been shown up to the previous chapter that it is theoretically possible to increase efficiency and power by inserting a canceling coil. The relationship between the ratio of CC to the main coupling and the transmission characteristics is an important indicator when considering applications that take CC into account. When the absolute value of the ratio of the main coupling ( $L_m$ ) to CC ( $L_c$ ) is  $\alpha$  (see equation (8)), the efficiency and the power shift are shown in Fig.5, respectively, where CC is set as  $L_c = L_{c12} = L_{c34}$  and the ratio of the main coupling ( $L_m$ ) to CC ( $L_c$ ) is  $\alpha$  ((8)). The transmission characteristics (Ideal.) obtained from the ideal device formulation and the experiment results using measured parameters (Exp.) are compared, and it can be confirmed that the peak efficiency and the frequency points for the minimum power are almost the same for different values of  $\alpha$  in the formulations and the circuit analysis.

$$\alpha = \left| \frac{L_c}{L_m} \right| \quad (8)$$

We can see that the efficiency and power characteristic curve itself shifts to higher frequencies than 45 kHz as the value of  $\alpha$  increases. Here, only the value of CC, i.e., the value of the molecule on the right side of equation (8), is changed. A larger value of CC relative to the main coupling shifts the efficiency curve more to the right. Due to the shift of the efficiency curve, the efficiency decreases at the 45 kHz resonance point and the efficiency improvement increases when CCC is achieved.

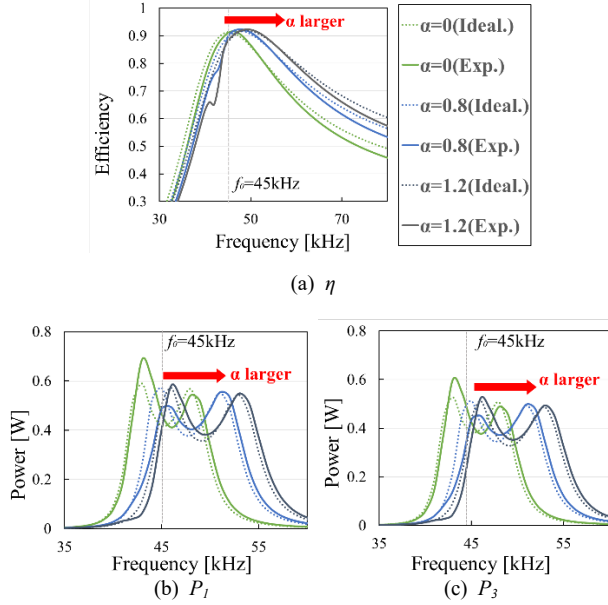


Fig.5. Change in characteristics depending on  $\alpha$

### C. Impact of CCC on Power

Fig.6 shows that the power value at the peak efficiency frequency decreases with the increase of  $\alpha$ .

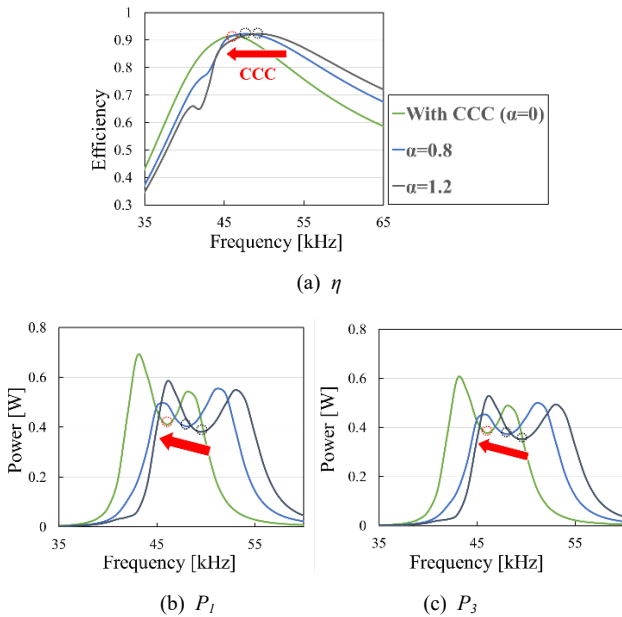


Fig.6. Changes in efficiency and power values under actual resonance

In order to improve the power transmission characteristics considering CC, we consider a method of tracking from the

defined resonant frequency to the frequency that is the central valley of the two peaks of power (higher than 45 kHz in this case; the direction of the arrow in Fig. 6) without changing the load connected to the Rx side. In this case, Fig.6(b) and (c) show that even if the efficiency is maintained as shown in Fig.6(a), the transmission power decreases. The case of  $\alpha = 0$  in Fig.5 and Fig.6 indicates the case where the exact CCC is achieved, which means that the power is improved in addition to the efficiency.

## IV. CONCLUSION

In this paper, CCC is formulated and its credibility is confirmed through analysis verification. In the formulation, it has been confirmed that the efficiency and power can be improved by manipulating the elements involved in CC by inserting the canceling coil. In the future, we will determine the input conditions of the power supply and the target values of the transmission power, and expand the verification of the effect of CCC.

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